Quantum Gravity Treatment of the Angel Density Problem

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Abstract

We derive upper bounds for the density of dancing angels on the point of a pin. It is found to be dependent on the assumed mass of the angels, with a maximum number $8.6766 \cdot 10^{49}$ of angels at the critical angel mass $3.8807 \cdot 10^{-34}$ kg.

1 Introduction

The problem of the number of angels that can dance on the head of a pin has been a major theological question since the Middle Ages [5].

According to Thomas ab Aquinas, it is impossible for two complete causes to be the causes immediately of one and the same thing, and since angels are such complete causes two angels cannot occupy the same space [2]. This can be seen as an early statement of the Pauli exclusion principle. However, this does not place any upper bound on the density of angels in a small area, since the size r of angels remains undefined and could possibly be arbitrarily small. There has also been theological criticisms of the assumption of angels as complete causes.

It should be noted that the original formulation of the problem did not refer to the head of a pin ($R \approx 1$ mm) but the point of the pin; this is the region that will be studied in this paper. The basic issue is the maximal density of active angels in a small volume.

One of the first reported attempts at a quantum gravity treatment of the angel density problem that also included the correct end of the pin was made by Dr Phil Schewe, who suggested that due to quantum gravity space is not infinitely divisible beyond a length scale of 10^{-35} meters. Hence, assuming the point of the pin to be one Ångström across (the size of a scanning tunneling microscope tip) this would produce a maximal number of angels on the order of 10^{50} [1].

While this approach does produce an upper bound on the density of angels, it is based on the thomist assumption of non-overlap. Since angels can be presumed to obey quantum rules when packed at quantum gravity densities, the uncertainty relation will cause their wavefunctions to overlap significantly even if there is a strong degeneracy pressure. If the non-overlap assumption is relaxed, this approach cannot derive an upper bound.

2 Quantum Gravity Treatment

A stricter bound based on information physics can be derived that is not based on overlap assumptions, merely the localisation of angelic information.

Assuming that each angel contains at least one bit of information (fallen / not fallen), and that the point of the pin is a sphere of diameter of 1 Ångström ($R =$

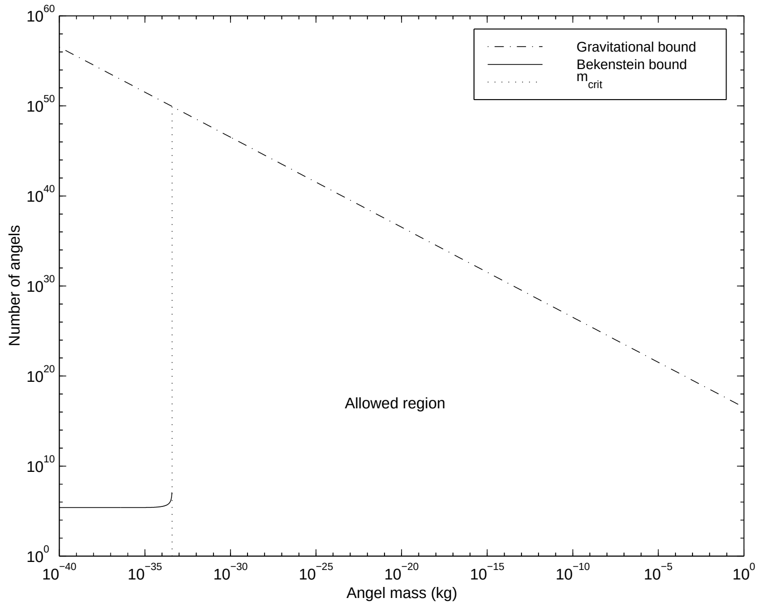

Figure 1:

10^{-10} m) and has a total mass of $M = 9.5 \cdot 10^{-29}$ kilograms (one iron atom), we can use the Bekenstein bound [3] on information to calculate an upper bound on the angel density. In a system of diameter D and mass M, less than kDM distinguishable bits can exist, where $k = 2.57686 \cdot 10^{43}$ bits/meter $\cdot$ kg [7]. This gives us a bound of just $2.448 \cdot 10^5$ angels, far below the Schewe bound.

Note that this doesn't take the mass of angels into account. A finite angel mass-energy would increase the possible information density significantly. If each angel has a mass m, then the Bekenstein bound gives us $N < kD(M + Nm)$. Beyond $m_{crit} > 1/kD \approx 3.8807 \cdot 10^{-34}$ kg this produces an unbounded maximal angel density as each angel contributes enough mass-energy to allow the information of an extra angel to move in, and so on.

However, if angels have mass then the point of the pin will collapse into a black hole if $c^2R/2G < Nm$ (here I ignore the mass of the iron atom at the tip) [4]. For angels of human weight (80 kg), we get a limit of $4.2089 \cdot 10^{14}$ angels. The maximal mass of any angel amenable to dance on the pin is $3.3671 \cdot 10^{16}$ kg; at this point there is only room for a single angel.

The picture that emerges is that for low angel masses, the number are bounded by the Bekenstein bound and increase hyperbolically as m_{crit} is approached. However, the black hole bound decreases and the two bounds cross at $m_{max} = 1/(4GkM/c^2 + kD)$, very slightly below m_{crit}. This corresponds to the maximal angel density of $8.6766 \cdot 10^{49}$ angels (see figure 1).

3 Dance Dynamics

If the angels dance very quickly and in the same direction, then the angular momentum could lead to a situation like the extremal Kerr metric, where no event horizon forms (this could also be achieved by charging the angels) [4]. Hence the number of dancing angels that can crowd together is likely much higher than the number of stationary angels.

However, at these speeds the friction caused by their interaction with the pin is likely to vaporise it or at least break it apart. In the case of charged angels at relativistic densities, pair-creation in their vicinity would likely cause the charge to dissipate over time [6], and charge transfer to the pin would also likely induce electromechanical forces beyond any material tolerances.

The uncertainty relation also imposes a limitation on the dance. Since the uncertainty in position of the angels by assumption are less than the size of the point $\Delta x \leq R$ we get that the uncertainty in momentum must be $\Delta p \geq \hbar/R$, and this leads to a velocity uncertainty $\Delta v > \hbar/Rm$. If $m = m_{crit}$ we get $\Delta v \geq 8.6766 \cdot 10^{59}$ m/s ($\gg c$), which shows that 1) the angels *must* dance with speeds near the velocity of light in order to obey quantum mechanics, 2) a full relativistic treatment is necessary and 3) that the precision of the dance must break down due to quantum effects. This can be used to rule out certain types of dance due to their high precision requirements.

4 Discussion

We have derived quantum gravity bounds on the number of angels that can dance on the tip of a needle as a function of the mass of the angels. The maximal number of angels $8.6766 \cdot 10^{49}$ is achieved near the critical mass $m_{crit} > 1/kD \approx 3.8807 \cdot 10^{-34}$ kg, corresponding the transition from the information-limited to the mass-limited regime. It is interesting to note that this is of the same order of magnitude as the Schewe bound.

Angel physics has so far mainly employed theological methods, but as this paper shows modern information physics, quantum gravity and relativity theory provide powerful tools for exploring the dynamics and statics of angels.

These bounds are only upper bounds, and do not take into account the effects of a finite number of available angels, degeneracy pressures if angels obey the Pauli exclusion principle as suggested by Aquinas or the theo-psychology of the angels themselves. The exact dance dynamics also clearly play a major role, and a full relativistic treatment of the dance appears as a promising avenue for further tightenings of the bounds.

References

[1] Reported in New York Times, November 11, 1997, Science Q & A: Dancing Angels by C. Claiborne Ray (http://www.nytimes.com/learning/students/scienceqa/archive/971111.html). Dr. Phil Schewe, presented his idea at a meeting of the Society for Literature and Science in 1995. It should be noted that the given density in the New York Times is wrong, as it did not count the area, only the length.

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